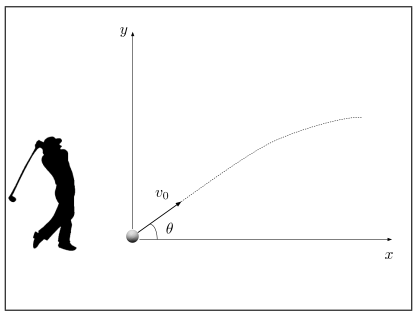
Project-Physics of the Golf ball

February 13, 2022

This project explores the trajectory of a golf ball through the air, and the factors in its construction effecting this trajectory. For instance, why does a golf ball have 'dimples' on it? The analysis can be extended to baseball, cricket, table-tennins, etc, without major modifications.
We shall break down the problem into steps, adding effects one at a time.

1. First, we ignore air resistance and compute the trajectory. This is a 'calibration', since the trajectory in absence of air is well-known. Following is the coordinate system to we use



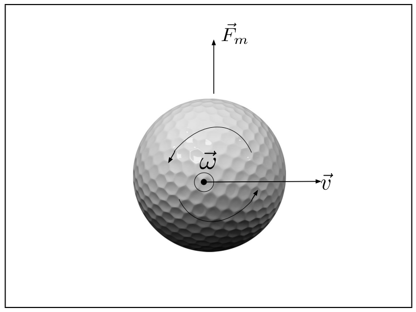
   The ball is hit such that it has initial speed v_0 and is hit at angle θ relative to the horizontal direction. Set up the equations of motion, assuming only gravity acts on the ball. Given the initial conditions and the physical constants appearing in the equations, identify natural velocity, time and length scales. Write the equations relating position, velocity and acceleration (components) in dimensionless form. Choose a (dimensionless) time increment, and using the RK2 algorithm, compute the position and velocity at instant of time upto the time the vertical coordinate becomes zero (the ball hits the ground. You will need a 'while' loop instead of a 'for' loop). Store the coordinates as lists to be plotted. Plot the trajectory (coordinate y vs coordinate x) for various angles, for initial velocity $v_0 = 70$ m/s (which is a typical golf ball initial velocity). Compare the results with exact analytical results known to you. For what initial angle does the ball have maximum range?

2. Next, we introduce air resistance. If the golf ball were smooth, the magnitude of the force of air drag would be well-approximated by the expression

$$F_D = C\rho A v^2 \tag{1}$$

   where v is the speed of the ball, $C = 1/2$, ρ is the density of air and $A = \pi r^2$ is the crossectional area of the ball. Introduce this force in the equations of motion. Using the same natural length, time and velocity scales as before (in absence of air resistance), write down the equations for the dimensionless coordinates and velocity components. Solve these equations once again (using the RK2 algorithm) for a variety of initial angles, and plot the trajectory of the ball, comparing it with the trajectory in absence of the drag force. For what angle is the range maximum? Assume that a standard golf ball has mass $m = 46$ g and radius $r = 22$ mm.
   Checkpoint: *In the equations, the dimensionless velocity dependent acceleration should have as coefficient $C\alpha$, where α is a dimensionless parameter given by $\alpha = (\rho A v_0^2)/mg$.*

3. **Magnus Effect**: If the ball is given a spin (when hit by the golf club), there is an additional force. This force arises because for a ball spinning about an axis perpendicular to direction of velocity, the speed of the air relative to the ball is different on opposite edges of the ball (why?). Depending on the direction of spin, the drag will either be higher or lower on the lower edge. If the ball is given a 'backspin', the force on the lower edge will be larger, such that the net vertical component of the force will be upward



The 'Magnus force' is given by the mathematical form

$$\vec{F}_m = a\,\vec{\omega} \times \vec{v} \tag{2}$$

Assuming that the direction of spin is perpendicular to the velocity and in direction illustrated ('backspin'), add this force to the equations of motion (assume that the spin does not change with time). Again, reduce equations to dimensionless form as before.
Checkpoint: *In the Magnus effect term, the dimensionless parameter $\beta = (av_0\omega)/mg$ should appear. For a typical backspin, $(a\omega/m) \simeq 0.25\ s^{-1}$* Again, determine the trajectory for various initial angles and compare with the previous results. For what initial angle is the range maximum? Do you observe an interesting curvature in the trajectory?

4. **Dimples**: Now, we put dimples on the ball. The introduction of dimples makes the air flow around the ball turbulent at higher speeds, which effectively decreases the drag coefficient C. Below speed of about 14 m/s, it is a constant (around 1/2). But above this speed, it decreases with speed. We assume the following form for the coefficient

$$C = \begin{cases} 1/2 & v < 14\text{m/s} \\ 7.0/v & v \geq 14\text{m/s} \end{cases} \tag{3}$$

Redo the analysis assuming this form of the drag coefficient. At what initial angle is the range maximum? How does the maximum range compare with that without dimples?